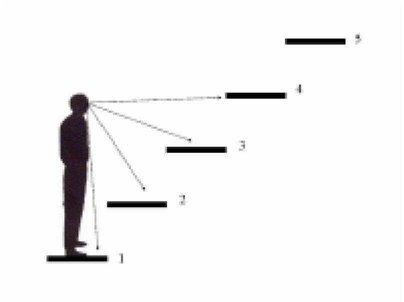
Relativity Simplified

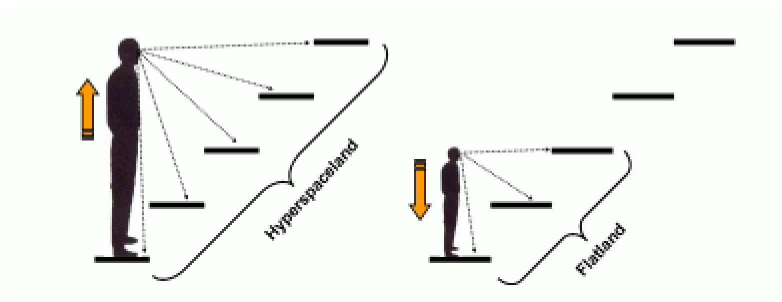
By R.F.J. van Linden

Relativity has always been taught using the so called Minkowski geometry, where the time dimension is markedly distinguished from spatial dimensions. It is possible though, to describe relativity using a more familiar Euclidean geometry where time and spatial dimensions are essentially identical in nature. Full articles on Euclidean relativity can be found on www.euclideanrelativity.com. Relativity is made easy in the description below that provides simple and intuitive explanations for a number of relativistic effects.
First, the concept of speed in 2, 3 and 4 dimensions is explained, meanwhile introducing the notion of speed in the time dimension. This is then used to show the mechanisms behind relativistic time dilation and length contraction. The last sections show why the speed of light remains constant in all situations, followed by a brief overview of conclusions from the main articles on Euclidean relativity.

1. Spatial dimensions and the time dimension



Imagine a man standing on a staircase with steps of about one third of his own height. He will be able to see the surface of the step he is standing on as well as the next two that are in front of him. The fourth step is harder to see, for his view on it is partly blocked (he may see it from the bottom or the front side). The fifth is even harder to see, or perhaps even invisible.

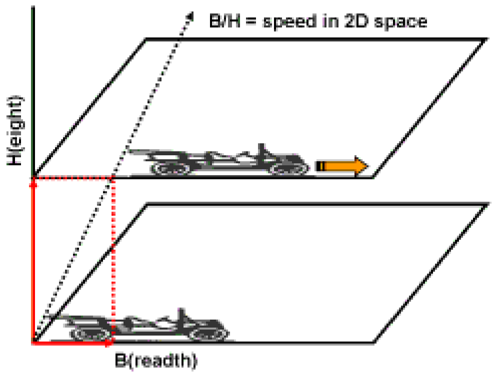


Suppose he eats something very nutritious and his body suddenly grows a full step in height. He is now able to see four steps but his view on the fifth step is partly blocked. That fifth step now looks exactly like the fourth step did before he grew. The looks of the other four steps are not really different from the three steps he used to see before.

He grows older and, like older people do, he shrinks in size. He shrinks a full two steps and now he can clearly see two steps but the third is partly blocked. That third step now looks identical to the original fourth.

We can translate this staircase into our space-time. The visible steps are our spatial dimensions and the partly visible higher step is our time dimension. The time step/dimension is actually just another spatial step/dimension but that only becomes clear when you move up and down in the dimensions. What you call a *spatial* dimension from your own "dimensional viewpoint" may be a *time* dimension from another observer's dimensional viewpoint. For the

man who grew tall, our 4-dimensional *space-time* is his 4-dimensional *space*, while he lives in a 5-dimensional space-time or "Hyperspaceland". For the man who shrunk, our 3-dimensional *space* is his 3-dimensional *space-time*. He lives in "Flatland".
What is it then that makes the time dimension appear to us as something different than a spatial dimension? That will be explained in the next section by showing that it is the time dimension that enables us to perceive speed. You can't do that with spatial dimensions alone.



2. Spatial speed and speed-in-time

We measure speed by dividing a covered *distance* by a time *duration*. Traveling one meter per two seconds, our speed is 1/2 m/s.

If we call the time duration (the seconds) a "length" in the dimension time we recognize that speed actually is a division of covered distance in a spatial dimension by covered distance in the time dimension.

A division of covered distance in two *spatial* dimensions, let's say dimension nr. 1 (breadth) and dimension nr. 3 (height), for us results merely in a "dimensionless" and rather meaningless number. However, for the old man whose size shrunk and lives in Flatland this division represents a spatial *speed* in his 2-dimensional world because dimension 3 is his time dimension. This implies that the position of his whole 2-dimensional environment must be changing in that third dimension, otherwise there would be nothing to divide the displacement in his spatial dimension nr. 1 by. In other words, he will only be able to measure it as a speed if his Flatland moves as a whole in the third dimension.

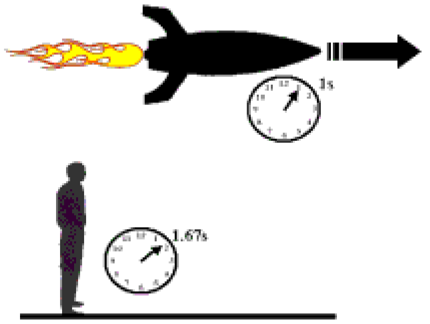
This displacement in the 3rd dimension looks to us like a spatial speed but the man who shrunk cannot see this as a spatial speed because he is not able to see this 3rd dimension as a spatial dimension. For him it intuitively *feels* like a progress in his time dimension.

Our intuitively felt "motion in time" actually is similar. Our 3D environment apparently moves as a whole in our time dimension too. And that enables is to measure speeds in our 3D spatial world.

So even if we do not measure any speed of objects in our spatial environment, e.g. if everything around us is standing motionless, that spatial environment still has a speed in the time dimension. But how can we express this speed in the time dimension in terms of a division of covered distances in two different dimensions? Seconds per second doesn't make sense because that actually uses the same dimension twice and will merely result in "1". And we don't have a spatial distance covered that could be used to measure that speed somehow.

For the man who grew tall after eating the very nutritious food, the answer is obvious. For him, the original time dimension turned into a normal spatial dimension and another, 5th dimension became his new time dimension. Our *speed in time* becomes a *spatial speed* in the 4-dimensional space of the tall man. If *he* wants to express that speed in his 4-dimensional space he divides the spatial distance covered by the time duration from his 5th dimension. So our inability to "see" our speed in time as a real motion is due to our inability to measure displacements in the fifth dimension that is used to calculate it. Our speed in time turns into a regular spatial speed as soon as one is able to perceive an extra dimension, like the tall man does.

But how then do we calculate the speed in time (that will henceforth also be referred to as "timespeed") for the tall man? Obviously it must be a displacement in his 5^{th} dimension divided by a displacement in yet another higher 6^{th} dimension. And so on, and so on. It's a recursive, or fractal-like system.



3. Spatial speed and timespeed are related

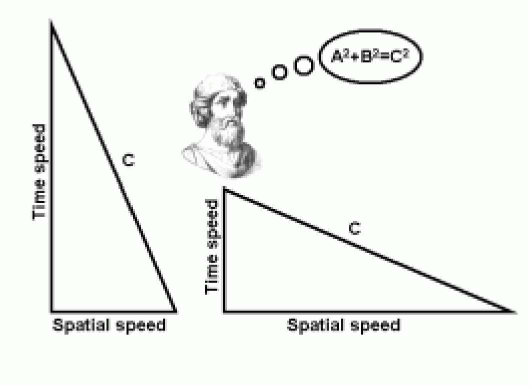
Einstein's theory of relativity predicts a limit in the speed that a thing can have. This limit is equal to the speed of light ***c*** and equals about 300.000 km/s.

The faster something moves, the slower time ticks away in that thing. If the speed of a rocket is for example $4/5^{th}$ of ***c*** (about 240.000 km/s) then one second on a clock in the rocket takes as much as 1.67 seconds according to the watch of somebody who stands still on the ground. So according to the man on the ground the clock in the rocket runs slow. He observes that the timespeed in the rocket is slower than his own.

Notice here that we quietly switched from a comparison of time *durations* (1 second versus 1.67 seconds) to a comparison of time *speeds*. It's straightforward to compare the time durations on the clocks at the man and in the rocket and express these in *seconds per second*. But in this case the comparison of the timespeeds can also be expressed in seconds per second because in that comparison the common and immeasurable fifth dimension that is used to calculate *both* speeds cancels out in the division. To see that, divide e.g. 1/x by 2/x. The result is 1/2 and the "x" has vanished.

We have seen that whenever an object's spatial speed goes up, its timespeed goes down and vice versa. When the spatial speed is zero, the timespeed must therefore be at its maximum. On the other hand when spatial speed equals its maximum, which is the speed of light ***c***, the timespeed actually has reduced to zero. For a clock in a rocket traveling at that speed it will take forever to tick away a second according to the man on the ground. Thus:

- Rule 1: zero spatial speed means maximum timespeed
- Rule 2: maximum spatial speed ***c*** means zero timespeed



A familiar formula can be used to express this relation between spatial speed and timespeed. It is Pythagoras' rule for rectangular triangles: $A^2+B^2=C^2$.

If we say that A is the spatial speed and B the timespeed then it appears that the sum of spatial speed and timespeed is always ***c*** (the speed of light) if we add them with Pythagoras' rule like in a triangle:

$$\textbf{(spatial speed)}^2 + \textbf{(timespeed)}^2 = c^2$$

This allows us to enhance our rules:

- Rule 1: zero spatial speed means maximum timespeed ***c***
- Rule 2: zero timespeed means maximum spatial speed ***c***
- Rule 3: the Pythagorean sum of both is always ***c***

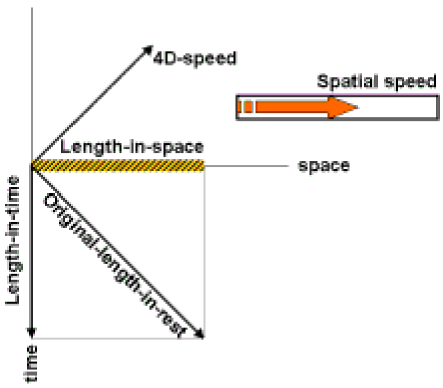
4. Length in space and length in time

Whenever something moves (fast) in space its length contracts in the direction of its motion. Furthermore the time *coordinate* is no longer the same in all points of the object but has *decreasing* values in points along the direction of its motion like in this picture:

As you can see, the time coordinate at the head constantly lags behind, compared to the coordinate at the tail.

We say that the points are *non-simultaneous* which basically means that adjacent points in the object are *in the past or future*, relative to each other !

The contraction of the moving object is a loss of length in the spatial dimension but the non-simultaneity of the adjacent points is in fact a gain of "length" in the time dimension. The usual length-in-time of an object in rest is zero. All its points then have the same time coordinate, i.e., they are all in the "now" relative to each other. The object is not contracted so its length-in-space is then also at its maximum.



Similar to the relation we saw above between the *speeds* in space and time, the *lengths* in space and time are also related according to Pythagoras' rule:

$$\textbf{(length-in-space)}^2 + \textbf{(length-in-time)}^2 = \textbf{(original-length-in-rest)}^2$$

To understand the mechanism behind this we will look at things from the perspective of the tall man again. Recall that the object's spatial speed goes up and its timespeed goes down, so the direction of its total speed actually *rotates* in 4 dimensions according to the tall man. Apparently the tall man sees that not only the object's speed rotates but along with it also its orientation in 4D. The object's length rotates *out of our 3D space* and gains a component in the fourth dimension, hence its length in our time dimension. What's left for us to see is the component of its 4D length in our 3D space, hence the contraction of its spatial length.

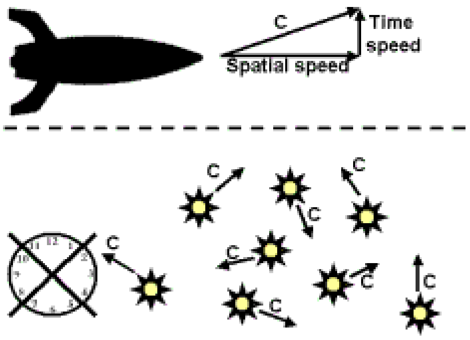
Another very nice interactive visualization of this effect was made by Adam Trepczynski on http://www.adamtoons.de/physics/relativity.swf (you will need the [Adobe Flash Player](#) browser addon to play it).

5. Speed of photons and speed of mass particles

Light rays travel at speed *c* and consist of millions and billions of tiny photons. These photons exist everywhere and travel in all possible directions in our 3D space.

The thing that all these photons have in common is that they all travel at *exactly* that same speed ***c***, no matter what direction they take and no matter who looks at them, even for an observer who travels at high speeds himself! An obvious question now is: "What is the timespeed of the photons?". Well, rule 3 that we derived in section 3 says that if they all travel at speed *c* in our space then their timespeed must always be zero. Time stands still for all photons. Always and according to every observer.

But if there is no timespeed in *any photon at all* we might just as well say that the whole time dimension does not *exist* for photons. Similarly, if everything in the universe would never ever move a single inch in space and it all would have started at a single point in space then everything would still be sitting at that same single point and you might as well say that space does not exist.



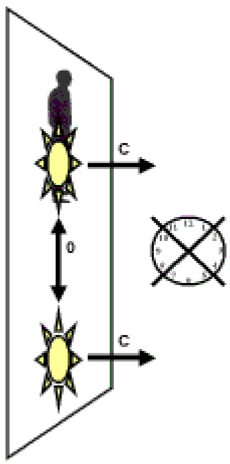
It's a bit more complex in reality. That's why this story is called a simplified version. In reality some properties of the photon that are related to its wave-nature do actually exist in the time dimension but to explain that here would make it all too confusing. So let's stick with the photon's particle-nature that has no clock. That photon lacks dimension number four so it is purely 3-dimensional and it always travels at speed ***c***.

Going back again to the speed of the rocket we see there that its combined speed, i.e. the Pythagorean sum of its spatial and timespeed is also ***c***. But we know that the timespeed was just a spatial speed for the tall man who ate the very nutritious food! That means that the rocket has a spatial speed ***c*** in four dimensions according to that tall man. Even stronger: *any* rocket, no matter what speed it has according to us, will have a spatial speed *c* in four dimensions according to the tall man (remember that this speed is *rotated* in four dimensions). If we now realize that this rocket actually consists of millions and billions of tiny mass particles then it is easy to see that any of these millions and billions of tiny mass particles travels at speed ***c*** in four dimensions according to the tall man. And they travel in all kinds of directions too because molecules, atoms, electrons, protons and so on travel criss-cross through each other.

6. Three-dimensional and four-dimensional worlds

It's a small step on the staircase to see the parallel between the photons and the tiny mass particles. The first apparently live in a 3-dimensional world while the latter live in a 4-dimensional world. Both travel at speed ***c*** in their dimensional environment. The 3-dimensional speed of the photons is measured by us using *our time* in the fourth dimension. The 4-dimensional speed of the mass particles is measured by the tall man using *his time* in the fifth dimension while we can only measure its spatial component using our fourth dimension.

We now take the place of the old man who shrunk. He sees only two spatial dimensions and one time dimension. But these together are the three dimensions that the photons live in. How does that translate in his world?
Whatever speed components the photons have in any of these three dimensions, they will always add up to ***c***, using Pythagoras. We put the old man on the back of one of these photons and imagine what he sees when another photon travels parallel to his photon. The two spatial dimensions of the old man form a plane (Flatland). We choose a nice symmetrical situation and assume that the flat space of the man stands rectangular to the direction in which his photon travels.



If the second photon is in the same plane and travels parallel with the first photon, the old man will think that the second photon stands still. On the other hand, if the second photon travels at an angle rectangular to the first photon, the old man on the back of the first photon will think that the second photon travels at speed ***c*** away from him in his plane.
Do not mix this up with the way *we* see photons move in *our* 3D space where we always see them travelling at speed ***c***, so *per definition* it is impossible for us to sit on the back of such a photon! The fundamental difference lies in the dimensions that are used to express the photon's speed. We use dimension nr. 4 in the denominator but the old man uses dimension nr. 3 so his result cannot be the same as ours.

A clever reader will now say: "Hey, this does not make any sense because if the second photon does not travel parallel to the first, it will immediately vanish out of the flat world of the old man and become invisible". This sounds logic, but what about the tiny mass particles traveling at high speeds in *our* spatial dimensions? They should immediately vanish from our 3-dimensional space too. To be more precise: they should actually vanish "in the past", because their timespeed is slower than ours. Still, we keep seeing them, so Nature somehow makes sure that we can always keep an eye on everything, no matter in what time zone it has "vanished".

The trick and only explanation is that the time dimension is fully contracted in our space along the direction of our own motion in it, like a sponge that is completely pressed flat in one direction. The same happens with the second photon that travels rectangular to the old man on the back of the first photon. Although it should "vanish" in the past of the old man (which for us is a regular spatial dimension), the old man keeps seeing it because for him this dimension is completely contracted into his flat space.

The bottom line is that the old man is able to see photons move at all speeds from zero to ***c*** in all kinds of directions in his flat world. So another parallel shows up with the way we see mass particles move in our space.

What does it mean?

To summarize what has been told up till now we can say that everything moves at speed ***c***, but you need to be "tall" enough to see that in all worlds. Your speed in time is equal to ***c***, because from your own point of view you yourself always stand still (your spatial speed with respect to yourself is zero). Furthermore, some particles, like the photons, obviously have one less dimension than others, like mass particles.

Did this story trigger you? If yes, you may want to read the full articles that are founded on a more scientific approach and deal with the "difficult" aspects that have been left out in this version. You really don't need to be an Einstein to follow it but some undergraduate background in physics is essential. See the links to the articles on www.euclideanrelativity.com.

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