Dimensions in Special Relativity Theory - *a Euclidean Interpretation**

R.F.J. van Linden
Smeetsstraat 56, 6171 VD Stein, NETHERLANDS
e-mail rob@vlinden.com, web http://www.euclideanrelativity.com

September 2005

Abstract

A Euclidean interpretation of special relativity is given wherein proper time τ acts as the fourth Euclidean coordinate, and time t becomes a fifth Euclidean dimension. Velocity components in both space and time are formalized while their vector sum in four dimensions has invariant magnitude c. Classical equations are derived from this Euclidean concept. The velocity addition formula shows a deviation from the standard one; an analysis and justification is given for that.

 PACS 03.30.+p.

1 Introduction

Euclidean relativity, both special and general, is steadily gaining attention as a viable alternative to the Minkowski framework, after the works of a number of authors. Amongst others Montanus [1,2], Gersten [3] and Almeida [4], have paved the way. Its history goes further back, as early as 1963 when Robert d'E Atkinson [5] first proposed Euclidean general relativity.

The version in the present paper emphasizes extending the notion of velocity to the time dimension. Next, the consistency of this concept in 4D Euclidean space is shown with the classical Lorentz transformations, after which the major *inconsistency* with classical special relativity, the velocity addition formula, is addressed. Following paragraphs treat energy and momentum in 4D Euclidean space, partly using methods of relativistic Lagrangian formalism already explored by others after which some Euclidean 4-vectors are established.

A simplified and popularized version is available that will get you in the 'right mood'. It can be found on the web at http://www.euclideanrelativity.com.

2 The Time Dimension

Minkowski interpretations of special relativity treat time differently from spatial dimensions, showing from the Minkowski metric where the time component is given the opposite sign. Some alternative interpretations (*e.g.* [1-4]) seek positive definite Euclidean metrics for space-time. Also in this article, the time dimension will be treated as a regular fourth dimension in Euclidean space-time.

If time is considered a fourth spatial dimension, then it must show properties similar to those found in the other three. In there we encounter properties like length, speed, acceleration, curvature *etc.*, expressed respectively as s, ds/dt, d^2s/dt^2, R^a_{bcd} *etc.* Of those properties, a single one can be measured relatively easily in the time dimension: the 'length' or timeduration Δt. That raises the question of how a hypothetical speed in time, let us call it χ, should be expressed mathematically. In [6], Greene has given a derivation of an expression that can be used as the velocity component in the Euclidean time dimension. Rewriting the usual Minkowski invariant

$$c^2 = (dct/d\tau)^2 - (dx/d\tau)^2 - (dy/d\tau)^2 - (dz/d\tau)^2 \tag{1}$$

into Euclidean form:

$$c^2 = (cd\tau/dt)^2 + (dx/dt)^2 + (dy/dt)^2 + (dz/dt)^2 \tag{2}$$

one arrives at the temporal velocity component

$$\chi = cd\tau/dt \tag{3}$$

This clearly defines τ as the coordinate for the fourth Euclidean dimension, and it says that the velocity components in all *four* dimensions involve derivatives with respect to t, which then can no longer represent the fourth dimension. It can only be an extra, fifth dimension, x_5 (provided we index the other four x_1, x_2, x_3, and x_4 respectively, with $\tau = x_4$). This fifth dimension is sometimes treated as a parameter in Euclidean approaches similar to special relativity, *e.g.* in [1,2], but here it will be treated as a genuine extra Euclidean dimension. A general expression for speed in the time dimension (henceforth refereed to as time-speed) is now:

$$\chi = cdx_4/dx_5 \tag{4}$$

while the scalar value of time-speed χ is

$$\chi = \sqrt{c^2 - v^2} \tag{5}$$

The general expression for spatial velocity components in 4D Euclidean space-time is

$$v_i = dx_i/dx_5 \tag{6}$$

3 Using Time-Speed in Special Relativity

It will be shown that the Lorentz transformation equations for length and time can be reproduced from the Euclidean context.

Maintaining orthogonality for all Euclidean dimensions, Eqs. (2) and (5) imply that the axes for the proper time dimension and the spatial dimension in the direction of the initial motion must have *rotated* for the moving object, as seen from the rest frame of the observer, in fact defining Lorentz transformations as rotations in SO(4). See also [1], where this is referred to as a Relative Euclidean Space-Time. In the approach that follows now, these axes will therefor (unlike in the Minkowski diagram) both rotate in the same direction, clockwise or counter clockwise, depending on the direction of the motion. The diagrams in Fig. 1 and Fig. 2 should illustrate this.

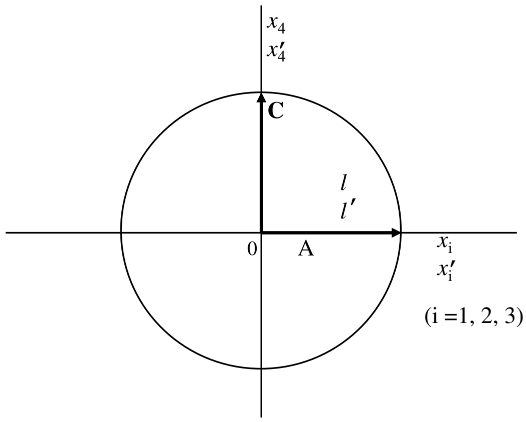

Figure 1: 4D representation of an observer at O and an object A, both at rest.

Figure 1 depicts an object A at rest together with an observer at O, also at rest. The horizontal axis shows both the spatial dimensions x'_i, $i = 1, 2, 3$, for the object A as well as the spatial dimensions x_i for the observer. The vertical axis shows both time dimensions with notation conform Eq. (2), so $x_4 = c\tau$. Due to object A being at rest, relative to the observer, the axes overlap. The circle is just a tool to better show the rotation that will be depicted in Fig. 2.

Definitions are as follows:

- Vector **C** indicates the 4D velocity, having magnitude c, of object A.

- Vector **V**, of magnitude v, and **X**, of magnitude χ, are the projections of this velocity **C** on, respectively, the spatial dimensions and the proper time dimension of the observer.

- l' indicates the proper length of object A in the spatial direction x'_i in the rest frame of object A (in this example l' is also set to c).

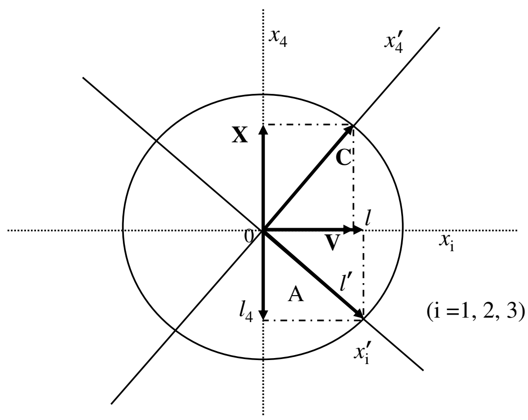

Figure 2: Object A in motion relative to observer. The dimensional axes of object A have rotated relative to the observer.

- l and l_4 are, respectively, the projections of this proper length on the spatial dimensions and the proper time dimension of the observer.

In Fig. 2, object A moves with speed v relative to the observer. This leads to a relative rotation of dimensions x'_4 and x'_i such that **V** is the projection of the original 4D velocity **C** of object A on the x_i axis of the observer at rest. The situation is examined at the instant where $x_i = x'_i = x_4 = x'_4 = 0$.

The Lorentz transformation equation for x is

$$x' = \gamma(x - vt) \tag{7}$$

where

$$\gamma = 1/\sqrt{1 - v^2/c^2} \tag{8}$$

but this factor can also be written as

$$\gamma = c/\sqrt{c^2 - v^2} = c/\chi \tag{9}$$

leading to

$$x' = c(x - vt)/\chi \tag{10}$$

At $t = 0$, the length of object A will be contracted, as measured by the observer, according to

$$x = x'\chi/c \tag{11}$$

so the contraction of length l can be written as

$$l = l'\chi/c \tag{12}$$

which shows that l, as measured by the observer at rest, is indeed the goniometric projection of the proper length l' on the x_i axis.

Arrow l_4 is the projected 'length' component of the moving object A on the proper time axis x_4 of the observer as a result of the rotation of the dimension x'_i. This length is the manifestation of the difference in proper time (the non-simultaneity) between the endpoints of object A in motion according to the Lorentz transformation equation for time:

$$t' = \gamma(t - vx/c^2) \tag{13}$$

and can be interpreted as a rotation 'out of space' of the proper length l' towards the negative axis of x_4. At $t = 0$ the proper-time difference between tail and head of arrow l will be

$$t' = -\gamma vl/c^2 = -lv/c\chi \tag{14}$$

From $l = l'\chi/c$ and $l_4 = l'v/c$ it follows that

$$l_4 = -ct' \tag{15}$$

which confirms that l_4 represents the proper-time difference in object A. The factor c results from the choice of units for space and time.

Summarizing, from the perspective of the observer, the proper length l' of object A is decomposed in the components l and l_4 according to:

$$l'^2 = l^2 + l_4^2 \tag{16}$$

and so is also the 4D speed c of the object decomposed in the components χ and v:

$$c^2 = \chi^2 + v^2. \tag{17}$$

Equation (16) thus combines Eqs. (7) and (13) into a single Pythagorean equation in four dimensions.

4 Relativistic Addition of Velocities

It appears that the Euclidean approach as used in the previous Section does not yield the same equation for relativistic addition of velocities as used in special relativity. Although this particular point may be a serious obstacle to the acceptation of this proposal, it obviously is necessary to point it out.

Figure 3 depicts a situation with three reference frames: a stationary unprimed frame x, a moving primed frame x' and a third, double primed frame x'' of an object that moves relative to both other frames, x and x'. Each frame has dimensional axes rotated relative to the other frames as a result of the relative motion.

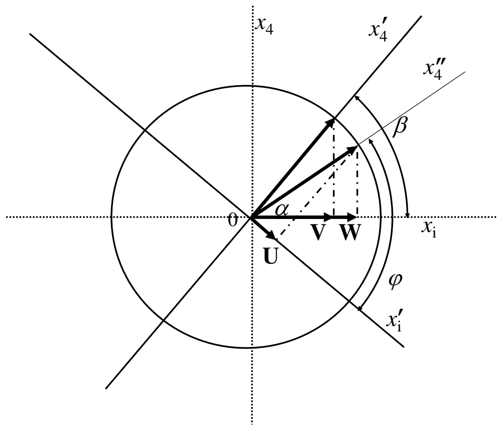

Figure 3: Relativistic addition of velocities in three reference frames, each with rotated dimensional axes relative to each other.

- Vector $\mathbf{V}$ of magnitude v is the spatial velocity of an observer with rest frame x' as measured by an observer with rest frame x.
- Vector $\mathbf{W}$ of magnitude w is the spatial velocity of a third object as measured by the observer with rest frame x.
- Vector $\mathbf{U}$ of magnitude u is the spatial velocity of that same object but now as measured by the observer with rest frame x'.

When u, v, and w are parallel, the classical relation between them is:

$$w = \frac{u + v}{1 + uv/c^2} \tag{18}$$

If we apply the approach as used consistently until now it yields the expression:

$$\begin{aligned} w &= c\cos(-\alpha) = c\sin(\frac{1}{2}\pi + \alpha) \\ &= c\sin(\beta + \varphi) = c(\cos\varphi\sin\beta + \cos\beta\sin\varphi) \\ &= u\sqrt{1 - v^2/c^2} + v\sqrt{1 - u^2/c^2} \end{aligned} \tag{19}$$

This expression is not nearly similar to the classical expression in Eq. (18).

Like Eq. (18), Eq. (19) still limits the speeds as measured by both observers to the maximum of c, which is also clear by inspection of the Figure. Some remarks will be made now on the probability of either of the equations to be the right one:

1. Equation (18) is in fact based on the universality of light speed and the basis for reasoning is that an object, *e.g.* a photon, having speed c for an observer in frame x will still have that same speed c for an observer in frame x'. This is one of Einstein's original postulates and also in this Euclidean approach it will still be maintained as a valid postulate, which essentially means that the photons velocity vector, as measured from the moving frame, must have rotated *along* with that frame. The third object, having speed w, as measured from frame x, is not a photon but a mass-carrying particle for which such a rotation apparently does not apply. It must therefor be emphasized that Eq. (19) for now may only be applied to mass-carrying particles.

2. Equation (18) shows a discontinuity that is unusual in physics. In Fig. 4, Eq. (18) is plotted for the situation where u always equals v.

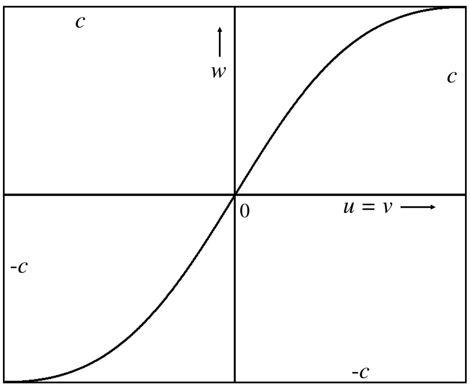

Figure 4: Graph of classical equation for relativistic addition of velocities.

With u and v nearing c, the resulting w will also near c, which is in accordance with the classical view. But if (as a matter of mathematical experiment) the range of u and v is extended beyond the maximum value of c then the plot looks like depicted in Fig. 5.

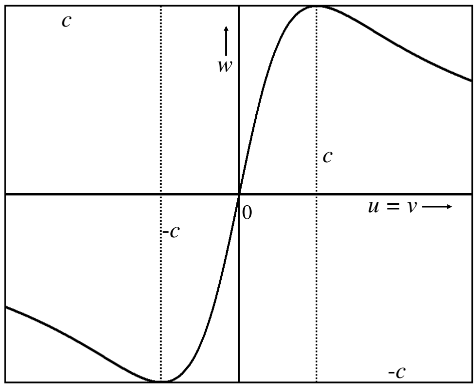

Figure 5: Classical graph for relativistic addition of velocities with hypothetical (superluminal) extensions.

The part from Fig. 4 can still be recognized but it is clear now that this actually forms part of a continuous function that extends beyond c. The part beyond $u = v = c$ may not be used, solely because the classical function is not defined, nor ever shown to be valid, for such superluminal extensions (actually the space-like quadrants in the classical light cone). This fact strongly suggests that the graph from Fig. 4 is an approximation of the real function.

Finally, both Eqs. (18) and (19) are plotted together in Fig. 6.

Equation (19) is almost identical for speeds below about $c/2$ but begins to deviate at higher speeds. The top of Eq. (19) corresponds to $u = v = c/\sqrt{2}$. From the circle diagram in Fig. 3 it shows that the time-speed of the object, as measured from frame x, then becomes zero. Equation (19) further shows *decreasing* values for w in situations where the values of u and v go beyond $c/\sqrt{2}$ (the frame of the moving object then rotates beyond $\pi/2$ relative to frame x). It turns out that in that case the corresponding time-speed for the object becomes negative. (This situation might be related to

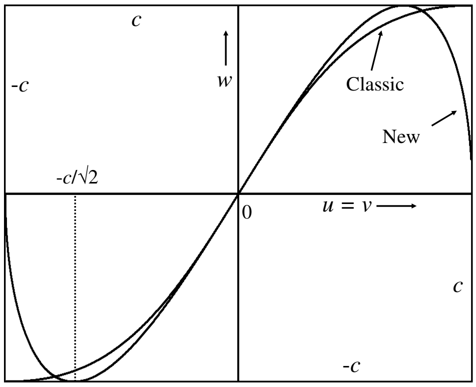

Figure 6: Classical [Eq. (18)] and newly derived graph [Eq. (19)] for relativistic addition of velocities plotted together for $u = v$.

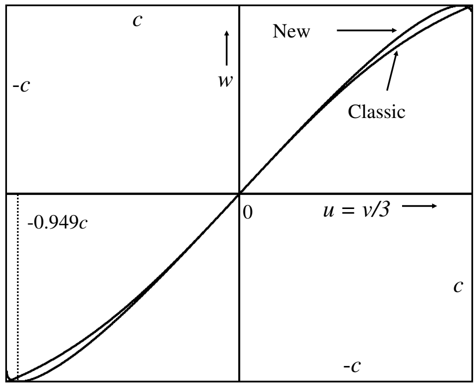

Figure 7: Classical [Eq. (18)] and newly derived graph [Eq. (19)] for relativistic addition of velocities plotted together for $u = v/3$.

anti-particles, running 'backwards in time'.).

The situation where u equals v gives the maximum possible deviation relative to the classical graph. Other ratios between u and v give (much) smaller deviations and the tops of Eq. (19) will shift outwards towards c as can be seen in Fig. 7 where the ratio between u and v equals 3:1. At a ratio 10:1 both plots are practically identical. Virtually all practical situations that require the velocity addition formula to be used exist under such circumstances, which indicates that a deviation from the classical graph is likely to remain unnoticed.

3. Some interpretations of Fizeau's experiment give rise to doubt concerning the correctness of Eq. (18). If Eq. (19) is used in the analysis of Fizeau's experiment done by Renshaw [7], it yields better results than Eq. (18), although still not within the margins as claimed by Michelson.

   The vast majority of experimental set-ups that are aimed at verification of relativity theory are using two reference frames. These experiments are not suitable for the verification of the velocity addition formula. One would have to use a set-up with three reference frames. At speeds on the order of 10^4 m/s the difference in resulting values between Eqs. (18) and (19) is on the order of 10^{-5} m/s, which might be noticeable using adequately accurate measuring devices.

A hypothetical case will now be used to show that Eq. (19) does not necessarily lead to causality conflicts as a result of the negative time-speeds that can occur.

A spaceship travels relative to Earth at speed $v_s = 0.9c$ and heads toward an asteroid that is at rest relative to Earth. The ship launches a missile at the asteroid at $v_m = 0.9c$ relative to the ship. An observer on the ship watches the missile destroy the asteroid. According to Eq. (19), an observer on Earth would see the missile traveling at only $0.7846c$ so the missile's spatial speed is *lower* than that of the spaceship. It seems therefor that this observer would see the ship hit the asteroid before the missile.

The explanation of this paradox can be found in the comparison of the proper times of all objects involved. We call the proper time for the spaceship τ_s and for the missile τ_m. For simplicity we set the space-time event of the launch at $t = \tau_m = \tau_s = 0$ and the distance between the spaceship and the asteroid at that moment at 0.9 light second (as measured by the observer on Earth).

The observer on Earth calculates time-coordinates of the impact (against the asteroid) using his own time t for the spaceship: $t_s = 1$s;

and for the missile: $t_m = 0.9/0.7846 = 1.147$s, so it seems as if the spaceship reaches the asteroid first. In 4D Euclidean space-time however the observer measures the time-speed χ_s of the spaceship as: $\chi_s = \sqrt{c^2 - v_s^2} = \sqrt{c^2 - (0.9c)^2} = 0.4359c$.

According to this observer the absolute value of the timespeed χ_m of the missile is $\chi_m = \sqrt{c^2 - (0.7846c)^2} = 0.62c$, but from the circle diagram (Fig. 3) it shows that we must now take the negative root so its value is $\chi_m = -0.62c$. Note that the cyclic nature of γ now also implies that in this situation γ has a negative value in $\tau_m = t_m/\gamma = t_m\chi_m/c$ for the missile.

We calculate the proper times at the moment of impact according to the observer on Earth for the spaceship: $\tau_s = t_s\chi_s/c = 0.4359$s; and for the missile: $\tau_m = 1.147(-0.62) = -0.7111$s.

In proper time the missile hits the asteroid before the spaceship does despite its lower spatial speed. Causality is therefor not violated. The missile runs backwards in proper time.

5 Relativistic Doppler Effect

Using the identity $\chi = \sqrt{c^2 - v^2}$ for the time-speed variable in the wavelength equation for the relativistic Doppler effect

$$\lambda' = \lambda_0\sqrt{\frac{1 + v/c}{1 - v/c}} \tag{20}$$

simplifies this expression to

$$\lambda' = \lambda_0(c + v)/\chi \tag{21}$$

It is possible to identify the individual contributions of the factors v and χ to the total Doppler effect by considering $\chi = c$ (which isolates the effect of the spatial speed) and $v = 0$ (which isolates the effect of the time-speed).

Setting $\chi = c$ results in:

$$\lambda'_v = \lambda_0(1 + v/c) \tag{22}$$

which is the regular equation for the acoustic Doppler effect with moving source and stationary receiver. Setting $v = 0$ results in:

$$\lambda'_\chi = \lambda_0 c/\chi \tag{23}$$

which simply makes the photon's frequency proportional to the time-speed of the emitting particle.

The relativistic Doppler effect can thus be interpreted as a combination of the normal 'acoustic' Doppler effect in space and a frequency shift that results from the lower time-speed.

6 Mass, Energy and Momentum

Figure 8 depicts a moving object with spatial velocity **V** of magnitude v, as measured by an observer at point L, at rest.

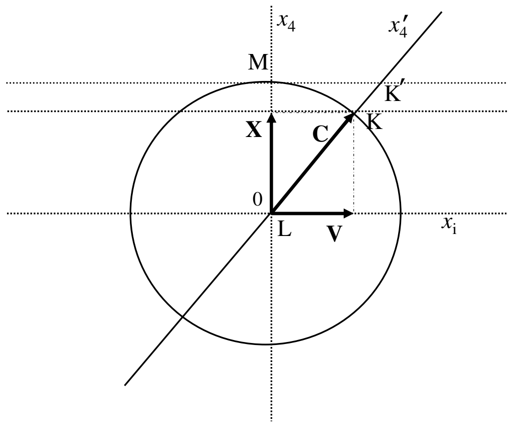

Figure 8: 4D velocity of magnitude c in x'_4 of an object at L. An observer at rest at L has velocity of magnitude c in x_4.

The vector sum of spatial and time-velocities reflects the four-velocities of the observer (along x_4) and the moving object (along x'_4). It follows naturally that the Lorentz invariant m_0c (m_0 is the rest mass) in the moving object A can be decomposed in

$$m_0^2c^2 = m_0^2\chi^2 + m_0^2v^2 \tag{24}$$

which, using the identities $E = \gamma m_0c^2$ and $p = \gamma m_0 v$, is equivalent to the classical equation

$$E^2/c^2 = m_0^2c^2 + p^2 \tag{25}$$

E being the total energy and p being the spatial momentum.

The components in the right part of Eq. (24) cannot simply be interpreted as, respectively, the object's momenta in the time dimension and the spatial dimension of the rest frame of the observer.

There is an obvious problem in the fact that the factor γ is involved in the expressions for E and p. If we multiply the factor γ^2 into all three elements of Eq. (24) we get:

$$\gamma^2 m_0^2 c^2 = \gamma^2 m_0^2 \chi^2 + \gamma^2 m_0^2 v^2 \tag{26}$$

which describes triangle LK'M (if m_0 is set to 1). This alternative form for Eq. (24) immediately shows the meaning of its components. They now correspond one to one with the components in Eq. (25): $\gamma m_0 c = E/c$, $\gamma m_0 \chi = m_0 c$, $\gamma m_0 v = p$. The factor $\gamma m_0 c$ is however not invariant under rotations in SO(4), while $m_0 c$ is. [Note that although $m_0 c$ is indeed Lorentz invariant from the perspective of the observer, its physical meaning in its own rest frame is the moving object's time-momentum. The same invariant value can be found in the rest frame of the observer (see also Fig. 9) but should then be read as $\gamma m_0 \chi$.] The Lagrangian formalism for this situation has been worked out by Montanus in [2]. The reader is therefore referred to this source for the detailed derivation. The generic principles used for such 5D situations (or more generally 4D with the addition of an extra parameter to keep track of the progress of the object along its world-line) appear in Goldstein [8]. The latter however uses the classical indefinite Minkowski metric as a basis for the development of the relativistic Lagrangian Λ where Montanus uses a positive definite metric like in this article. A short overview of the main equations is given here.

In agreement with classical mechanics it is assumed that the variation according to Hamilton's principle:

$$\delta I = \int_{x_{5(1)}}^{x_{5(2)}} \Lambda(x_\mu, u_\mu) dx_5 \tag{27}$$

is an extremum, where $u_\mu = dx_\mu / dx_5$. The corresponding Euler-Lagrange equations of motion are:

$$\frac{\partial \Lambda}{\partial x_\mu} - \frac{d}{dx_5}(\partial \Lambda / \partial u_\mu) = 0 \tag{28}$$

leading to a possible relativistic Lagrangian for a free object in the absence of a forcefield (so the potential energy equals zero):

$$\Lambda = m_0 u_\mu u^\mu \tag{29}$$

which equals, as a result of the universal velocity magnitude c for the free particle in 4D space-time:

$$\Lambda = m_0 c^2 \tag{30}$$

The latter is to be interpreted as the 'kinetic energy' of the particle in four dimensions, which is a fundamentally different concept than kinetic energy in three dimensions. It corresponds to the total energy of a particle at rest. Other solutions for Λ are possible but the essential element is that any solution is a constant in 4D space-time.

The relativistic Lagrangian Λ shows that the factor γ in Eq. (26) must be a result of our confinement to a 3D subspace of 4D space-time. In order to maintain conservation laws for energy and momentum, while only being able to measure their 'projections' to our 3D space, the factor γ is an artificial necessity. It vanishes for a hypothetical observer with full 4D observational skills, who measures the object's speed and energy as constants.

7 Transformation of Energy and Momentum

The generic transformation equations for energy and momentum depend indirectly on the equation for relativistic addition of velocities. Because a new one replaces this equation, it is necessary to rework the transformation equations for energy and momentum as well.

Figure 9 depicts an object moving with velocity **W** of magnitude w relative to frame x and velocity **U** of magnitude u relative to frame x'.

(please refer also to Fig. 3 and the definitions given there)

- $E = \gamma(w) m_0 c^2$ is the energy of an object that moves with velocity **W** of magnitude w relative to frame x and measured in frame x.
- $E' = \gamma(u) m_0 c^2$ is the energy of that same object moving with velocity **U** of magnitude u relative to frame x' and measured from frame x'.
- Frame x' moves with velocity **V** of magnitude v relative to frame x.
- $\gamma(u) = 1 \Big/ \sqrt{1 - u^2/c^2}$

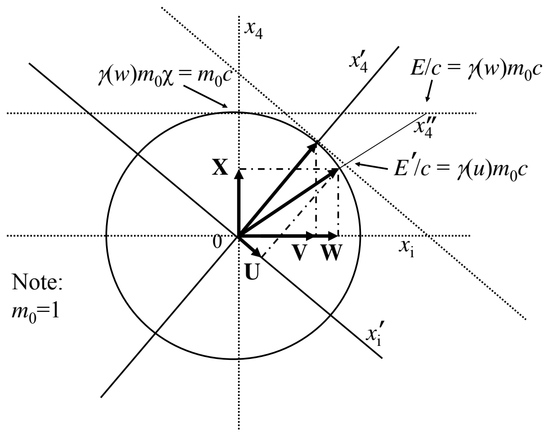

Figure 9: Generic transformation of energy and momentum in three reference frames with rotated dimensional axes.

- $\gamma(v) = 1 \Big/ \sqrt{1 - v^2/c^2}$
- $\gamma(w) = 1 \Big/ \sqrt{1 - w^2/c^2}$

For energy this leads to a generic transformation equation

$$E/E' = \gamma(w)/\gamma(u) \tag{31}$$

which can be written in different forms using Eq. (19). With $u = 0$ this reduces to the classical form:

$$E/E' = \gamma(v) \tag{32}$$

For momentum a generic transformation equation is

$$p/p' = wE/uE' \tag{33}$$

where:

- $p' = \gamma(u)m_0u$ is the momentum of the object as measured from frame x'.
- $p = \gamma(w)m_0w$ is the momentum of the object as measured from frame x.

8 Euclidean Four-Vectors

The traditional Minkowski line element with metric $(+1, -1, -1, -1)$ is:

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2 \tag{34}$$

where $ds = cd\tau$. Four-vectors with the Euclidean metric $(+1, +1, +1, +1)$ as used in the previous Sections use the 4D velocity of the moving object and 4D Euclidean distances as invariants, which is in fact the essence of Eq. (2):

$$c^2 = v_1^2 + v_2^2 + v_3^2 + \chi^2 \tag{35}$$

Multiplication with $dt^2 = dx_5^2$ yields (recall that $\chi = cd\tau/dt$):

$$c^2dt^2 = dx_1^2 + dx_2^2 + dx_3^2 + c^2d\tau^2 \tag{36}$$

where the factors $c^2d\tau^2$ and c^2dt^2 from Eq. (34) have switched roles.

The Euclidean metric thus gives rise to four-vectors for position, velocity and energy/momentum:

Euclidean	Minkowskian
$(x_1, x_2, x_3, c\tau)$	(x_1, x_2, x_3, ct)
(v_1, v_2, v_3, χ)	$\gamma(v_1, v_2, v_3, c)$
$(m_0v_1, m_0v_2, m_0v_3, m_0\chi)$	$(p_1, p_2, p_3, E/c)$

Equation (36) is not really new. It is merely Eq. (34) written in a different form, with as a main input the definition of χ, being the time-speed of an object as measured by an observer at rest, which has three effects:

- It creates a new invariant c, being the universal magnitude of the 4D velocity of an object.
- It provides a Euclidean basis for the definition of vectors in the direction of the time dimension.
- It enables these new vectors to be summed with existing vectors in the spatial dimensions.

In general, the new Euclidean four-vectors can be derived from the Minkowski four-vectors by using the time component in the Minkowski four-vector as the invariant (the vector sum) for the new four-vector. It is essentially doing Pythagoras "the other way around", *i.e.*, calculating the hypotenuse from the rectangular sides, instead of calculating a rectangular side from the hypotenuse and the other rectangular side (refer to [9] for a detailed treatment of Minkowski and Euclidean four-vectors).

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